NMR diffusometry and the short gradient pulse limit approximation

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Abstract

In NMR diffusometry, one often uses the short gradient pulse (SGP) limit approximation in the interpretation of data from systems with restricted diffusion. The SGP limit approximation means that the gradient pulse length, δ, is so short that the spins do not diffuse during the pulse duration, but this condition is rarely met. If the length scale of the pores corresponds to the molecular mean square displacement during the gradient pulse, the measured echo intensities become a function of the gradient pulse length. Here, we have studied highly concentrated emulsions to show how the length of the gradient pulse influences NMR diffusion experiments. We have focused on molecules confined to one pore and molecules that can migrate through the porous system. For the former the echo decays give smaller pores than the actual case and for the latter we show large changes in echo decay depending on the gradient pulse length, everything else being equal.
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1. Introduction

In NMR diffusometry one often uses the short gradient pulse limit approximation in the interpretation of data from the application of the technique to systems with restricted diffusion such as in concentrated emulsions, but the quality of the information obtained when using the approximation is a matter of some uncertainty [1–3]. The diffusion induced echo attenuation from such systems is conveniently considered in the scattering wave vector approach, with q as a measure of the gradient pulse area. A plot of the measured intensity versus q gives information about the porous morphology. To first approximation, the SGP limit approximation means that the gradient pulse length, δ, is so short that the spins do not diffuse during the gradient pulse, a condition that is rarely met. For porous systems the condition for the validity of the SGP limit approximation depends on the porous length scale through:

$$\delta \ll \frac{a^2}{D_0}, \tag{1}$$

where a is the pore radius and D_0 the bulk diffusion coefficient of the liquid contained in the porous system. When the condition is fulfilled, then the position of the spin-bearing molecule is probed at a specific place in the pore. When this condition is not fulfilled the analysis of NMR diffusion data yields a pore size smaller than the actual case [4].

Several authors have discussed the problem associated with the SGP approximation. Blees considered the problem of diffusion between two plates by numerically solving the Bloch–Torrey equation (5). Mitra and Halperin [6] approached the problem with a center-of-mass distribution function, which provides a useful and intuitive feeling for the origin of the effects observed for finite gradient pulses. Linse and Söderman [7] performed Brownian dynamic simulations for spins in planar, cylindrical and spherical geometries, and compared the simulation data with SGP results.

In 1996, a method was proposed by Caprihan et al. [8] to add multiple narrow pulses, each being in the small gradient pulse limit and Callaghan [4] presented a useful mathematical formalism for this approach in 1997. In 1999, Codd and Callaghan presented results, based on Caprihan's work from 1996 and Callaghan's work from 1997, for practically all gradient wave forms [9]. Finally,

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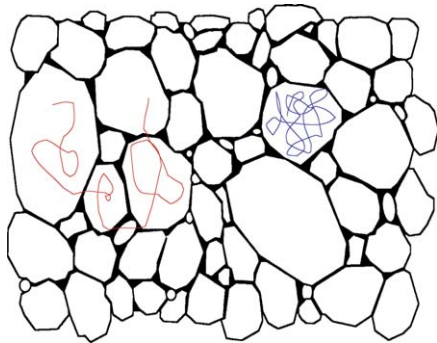*E-mail address:* carin.malmborg@fkem1.lu.se (C. Malmborg).

Fig. 1. A schematic picture of a concentrated emulsion. The deformed water droplets are surrounded by a thin oil-film. On account of its finite solubility in the oil-film, a water molecule can diffuse between the droplets (illustrated in the left part of the figure), while an ion (on the right-hand side of the figure) is confined to one droplet during the diffusion time.

Barzykin [10] presented an approach based on the eigenmodes of the diffusion propagator in restricted geometries 1999. All this work dealt with molecules confined to one pore with reflecting wall boundary conditions.

When the diffusion is not restricted to one pore, the problem is more complicated. To treat this case, Callaghan and Coy [11] developed a pore hopping theory based on the SGP limit in 1992.

In this work we investigate the effects due to finite gradient pulses experimentally. We work with highly concentrated water-in-oil emulsions containing 95 wt% water solution (with 0.2 M added salt for stabilization purposes) 3.5 wt% heptane as oil phase and 1.5 wt% Brij92 as surfactant. This type of emulsion, sometimes called foam emulsion or emulsion gel, is important in many technical applications such as in food products, in cosmetic, and pharmaceutical formulations and in emulsion explosives, to name just a few of the many application areas. But they are also useful as model systems in developing NMR diffusometry as applied to porous systems [12,13]. They have a droplet size of a few μm and are stable for at least 15 h, which is adequate time for performing the NMR measurements. Moreover, with this system we may investigate both effects for molecules confined to a droplet and for molecules that may hop from one pore to the next (cf. Fig. 1). We will show how the NMR diffusometry approach yields important information about the diffusion of the various components in these soft matters and how the gradient pulse length affects the result.

2. NMR diffusometry

NMR diffusometry is a technique based on two identical gradient pulses of duration δ and amplitude g a distance Δ apart on either side of the 180° pulse in a Hahn echo or, in the case of stimulated echoes, between the first and second 90° pulse, and after the third 90° pulse [1–3]. The normalized spin echo intensities can be written as:

$$E_{\Delta}(q) = \int \int \rho(z_0)P(z_0|z,\Delta)e^{i2\pi q(z-z_0)} \, \mathrm{d}z\mathrm{d}z_0, \qquad (2)$$

where $\rho(z_0)$ is the initial normalized spin density and $P(z_0|z,\Delta)$ is the probability density of finding a spin at position z after a time Δ, if it was originally at z_0, Δ is the diffusion time, i.e., the time over which the root mean square displacement is monitored. q is defined as $\gamma g\delta/2\pi$, where g is the magnetogyric ratio of the studied nucleus. Eq. (2) is valid for infinitely short gradient pulses, such that no diffusion of spins occurs during the application of the pulse. Apart from this limitation, Eq. (2) is valid for all types of diffusion processes (if relaxation is uniform throughout the pore space).

For unrestricted diffusion, where the propagator is Gaussian, Eq. (2) gives the Stejskal–Tanner equation:

$$E_{\Delta}(q) = \exp\left(-4\pi^2q^2\Delta D\right), \qquad (3)$$

where D is the self-diffusion coefficient. A useful property of NMR diffusometry is that the mean square displacement (MSD) may be obtained by expanding Eq. (2) to low values of q:

$$\lim_{q\to 0}E_{\Delta}(q) = 1 - 2\pi^2q^2\langle z^2\rangle. \qquad (4)$$

3. PGSE NMR in concentrated emulsions

The highly concentrated water-in-oil emulsions we have used contain 95 wt% water with a thin oil film around the water droplets. Fig. 1 serves as a background for the following discussion. The droplet diameter is about 3 μm and the oil film 10 nm. The water molecules diffuse inside the water droplets, but they may also pass the oil film and move to other droplets by diffusing through the film, thus essentially performing a random walk between droplets [14].

Consider a liquid with self-diffusion coefficient D in a spherical pore with radius a. If the diffusion time $\Delta \ll a^2/D$ the spin will show free Brownian motion behavior, except of course those spins placed close to the pore wall. For intermediate time-scales such that the spins have not left the droplets, but $\Delta \gg a^2/D$, the motion is restricted, and the labeled spin may be found anywhere in the pore after the diffusion time. For longer time-scales, the spins diffuse between the pores, and may be found anywhere in the sample. If the structure is well-ordered, at least locally, diffraction peaks may be observed in a plot of the measured echo intensities versus q. The inverse of the q value of the top of the diffraction peak equals the characteristic distance, from one droplet to another, which is the diameter of the pore. The echo profile peaks are quite broad and hence the evaluation is

not very exact. This fact may have different causes. The water droplets are polydisperse in size and/or the emulsion is divided in big domains with different sizes of the droplets in each domain. When the signals are added together both of these situations would give broad peaks with a mean droplet size.

For those molecules that are confined to one pore during the experiment, and for diffusion times such as that $\Delta \gg a^2/D$ we can write (1)

$$\langle z^2 \rangle = \frac{2}{5} a^2, \tag{5}$$

and thus calculate the pore radius, a from the MSD.

4. Results and discussion

4.1. Emulsion stability

Since the structure of an emulsion is in a non-equilibrium state it is important that there are no changes in the structure during the experiment. As noted above, the echo attenuation contains structural information, which can be used to investigate the emulsion stability. Thus, echo profiles have been measured before and after each experimental series in which δ and Δ is varied. Fig. 2 shows that the emulsion is stable during the measurement (15 h) and that the emulsion droplet size is about 3.3 μm ($1/q_{max}$).

4.2. Molecules confined to one droplet

As noted in the introduction, theoretical work has shown that the pore size, if determined from the diffusion induced echo decay of molecules confined to a pore, is smaller then the actual size if the SGP approach is used [4–7,9,10]. In treating the problem, Mitra and Halperin [6] made use of the center-of-mass (com) av-
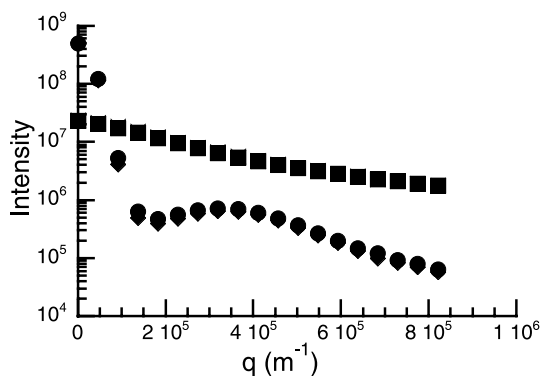
erage of the Brownian trajectories during the gradient pulse. In the SGP approximation, a particle is position encoded instantaneously. With finite length gradient pulses, the particle is encoded for a position that is the com of the Brownian motion trajectories during the pulse. Consider the case when $\delta > a^2/D_0$. Then the molecules sample most of the pores and the com of the Brownian trajectories is close to the center of the pore. During the second "read" pulse, identical to the first one, the same situation applies, and if the MSD is interpreted as the MSD of the com in the interval between the pulses, the pore is conceived as being much smaller than its actual size. Our experimental results are inline with this. Fig. 3 shows the echo attenuation curves for the tetramethyl ammonium ion plotted vs. q^2 with the pulse length, δ, increasing from bottom to top. It is evident in Fig. 3 that the initial slopes depend on the value of δ. In Fig. 4 we show the MSD data, calculated from (Eq. (4)) using data at low q values vs. δ (upper curve). Also included are results of Brownian simulations of random walk within a sphere of radius 1.8 μm with reflecting walls (lower curve). The simulated MSD
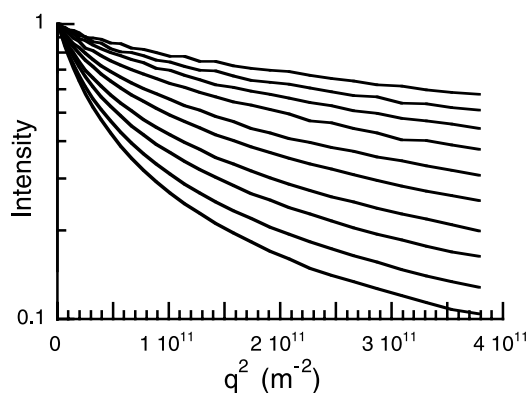
Fig. 3. Experimental echo intensity vs. q^2 for tetramethyl ammonium ions. δ is varied from 1.5 to 50.0 ms from bottom to top, for values see text. ($\Delta - \delta/3$) is kept constant at 200 ms.

Fig. 2. An example of echo intensity vs. q plot. Water before the experiments, ●; water after the experiments, ◆; salt before the experiments, ■; salt after the experiments, ✖. Water is diffusing between droplets, while salt is not (cf. Fig. 1). The inverse of the peak maximum is related to the droplet diameter.
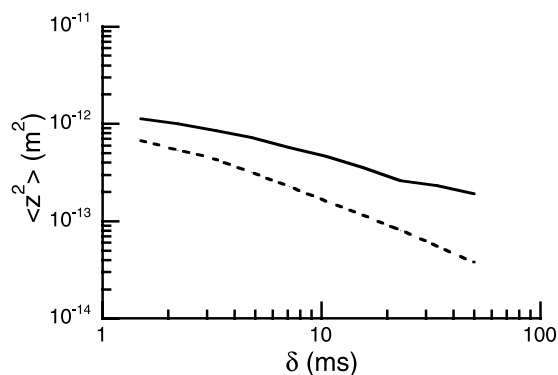
Fig. 4. Mean square displacement, $\langle z^2 \rangle$, vs. δ for experimental (solid line) and simulation data (dotted line).

data is displaced to lower values when compared to the experimental data. We attribute this to the uncertainty in the value of sphere radius and the fact that in the experimental system both the shape and size are polydisperse.

From the experimental data in Fig. 4 it is clear that the determined MSD decreases by one order of magnitude when the pulse length is increased from 1.5 to 50 ms. To give an estimate for the magnitude of the effect, we have chosen to calculate the pore radius for the case when $\delta = 10$ ms. During 10 ms the MSD of a molecule with a diffusion coefficient of 10^{-9} m^2 s^{-1} is roughly 4 μm. Under those conditions we obtain from Eq. (5) a pore radius of 1 μm, which can be compared to the "correct" value, which is close to 2 μm (see above).

Before leaving this section we note that the so-called Gaussian phase approximation, in which the accumulation of phase shifts during the field gradient pulses is assumed to be Gaussian [15] can be used to accurately describe the MSD of molecules confined in a sphere with reflecting walls. Plotted in Fig. 5 is the MSD for molecules in a sphere with reflecting walls as obtained from the initial slopes of echo intensities vs. q for both simulated data and data from the relevant GPD equation. As is evident in Fig. 5, the GPD results at low q values give a relevant description of the MSD, and can thus be used to determine sizes of confining geometries. The reason for this is presumably that for small phase shifts, such as those at small values of q, the GPD results give an accurate enough description of the initial echo intensities. The GPD equation is after all the first in a series expansion, which if enough terms are included, can be made to well predict PGSE data with finite gradient pulse lengths at arbitrary q values [10]. The GPD approach cannot describe the diffraction effects observed at larger q values for molecules diffusion in well-defined geometries (such as in spheres) [1] since the echo intensities predicted by the GPD approach are sums of exponentials.

4.3. Molecules migrating between pores

Fig. 6 shows the echo decays of diffusing water molecules in a concentrated emulsion for different pulse lengths. Diffusion in a porous system consisting of well-defined pores separated by narrow throats can in the SGP approximation be analyzed using the pore hopping formalism developed by Callaghan et al. [1,11]. A condition for this formalism to be applicable is that the molecules stay long enough within one pore to have equal probability of being anywhere in the pore, before moving on in the porous network during the diffusion time Δ. For a system consisting of pores with size a, distance b between the centers of neighboring pores, bulk diffusion coefficient D_0 and long-range diffusion coefficient D_∞ within the porous network, the condition is

$$\frac{a^2}{D_0} \ll \frac{b^2}{D_\infty}. \qquad (6)$$

When this condition is fulfilled the propagator is a convolution of the autocorrelation function of the shape of a single pore $\rho(z)$ and the micro-structural correlation function $L(Z)$, describing the relative positions of the pores in space, weighted by the diffusion envelope $C(Z,t)$. The echo attenuation, which is the Fourier transform of the propagator, can be written

$$E(q,t) = |FT\{\rho(z)\}|^2 [FT\{L(Z)\} \otimes FT\{C(Z,t)\}], \qquad (7)$$

where t is the effective diffusion time (equal to Δ in the SGP limit, but for finite pulses t can be taken to be $(\Delta - \delta/3)$). At such values of the diffusion time t, that the molecules have displacements on the same length scale as some characteristic distance of the structure, a
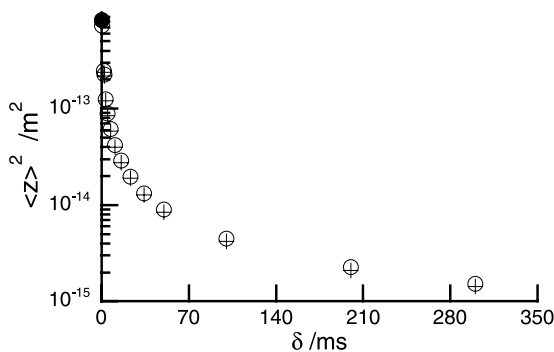

Fig. 5. Mean square displacement, $\langle z^2 \rangle$, vs. δ, obtained from Brownian dynamics simulations (+) and Gaussian phase distribution calculations (○). The MSDs were obtained from the initial slopes of echo intensities vs. q plots. The radius of the sphere was 1.5 μm and the diffusion coefficient of the spins in the sphere was $D = 2 \times 10^{-9}$ m^2 s^{-1}. Also included is the short gradient pulse results (●).
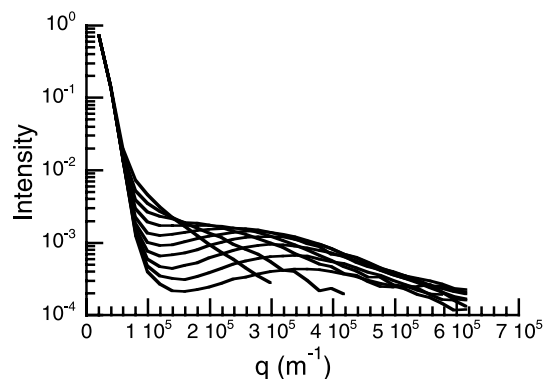

Fig. 6. Experimental echo intensities vs. q for the water peak. δ is varied from 1.5 to 50.0 ms from bottom to top at the minima, for values see text. $(\Delta - \delta/3)$ is kept constant at 200 ms. Each curve consists of 32 data points.

plot of $E(q,t)$ may display diffraction-like features, cf. Figs. 2 and 6.

For the case of finite gradient pulses no expression analogous to Eq. (7) has been published. A qualitative estimate of the effects can be obtained with the center-of-mass propagator introduced by Mitra and Halperin [6] and discussed above.

In the following section we will discuss Fig. 7, which illustrates the effect of increasing the pulse length when the molecules on average travel through a few pores during the diffusion time, i.e.,

$$t \geqslant \frac{b^2}{D_\infty}. \tag{8}$$

This case corresponds to the diffusion of water in the emulsion investigated here on the 100 ms time-scale. At such diffusion times, the initial slope of a plot of $E(q,t)$ is given by D_∞. The initial slope for a series of curves calculated for different δ will coincide if t is constant.

As noted above, the condition for the SGP-approximation is

$$\delta \ll \frac{a^2}{D_0}. \tag{9}$$

When this condition is fulfilled a particle can be labeled for a position anywhere within the pore space, see Fig. 7A, and the signal is given by Eq. (7). At longer δ, when

$$\delta \approx \frac{a^2}{D_0} \tag{10}$$

the particle is not labeled for a position close to a pore wall, but rather for a position closer to the center of the pore (the com of the Brownian trajectories during the pulse), which is shown in Fig. 7B. The longer δ, the more particles are labeled for the center. In the limit where all particles are labeled for the pore center, Eq. (7) reduces to

$$E(q,t) = \mathrm{FT}\{L(Z)\} \otimes \mathrm{FT}\{C(Z,t)\}. \tag{11}$$

This limit is not very likely to be encountered in a real experiment, but a decreasing influence of the first factor in Eq. (11) is expected in a water-filled porous system with pores on the 1–10 μm length scale. It is thus possible to reduce the influence of the pore shape on $E(q,t)$ by a proper adjustment of δ. In analogy with conventional scattering theory this corresponds to the isolation of the structure factor. In the limit when

$$\delta \gg \frac{b^2}{D_\infty} \tag{12}$$

the molecules are labeled for any position in the pore space, see Fig. 7C, and the signal approaches a Gaussian corresponding to a propagator with a width given by D_∞. Since δ must be smaller than t, the value of t determines if the Gaussian limit can be reached.
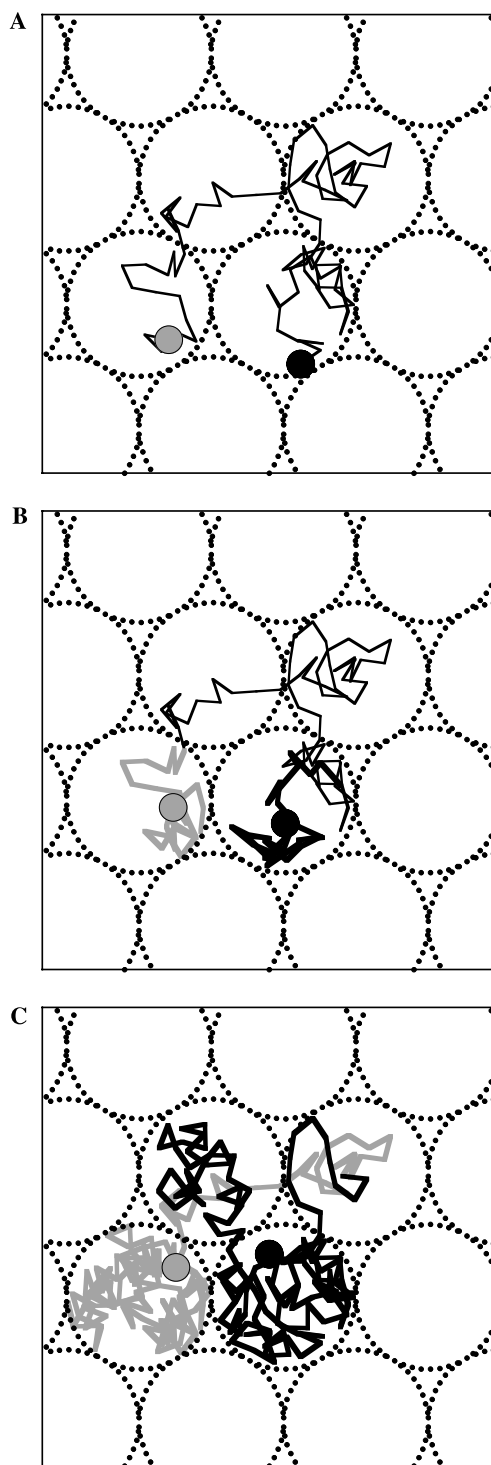

Fig. 7. The effect of increasing the pulse length when the molecules on average travel a couple of pores during the diffusion time. δ is increased from panel A to C, while $(\Delta - \delta/3)$ is kept constant. The thick part of the trajectory in each panel corresponds to the diffusion path during the gradient pulse. The center-of-mass average of the particle trajectory during the label and read pulses are marked with gray and black dots.

The finite gradient pulse thus has a "coarse-graining" effect. When the finite pulse length is such that the spin bearing molecules have a finite probability of actually

leaving a pore during the pulse, the information about the pore is lost. In fact, for sufficiently long gradient pulses, the porous nature of the sample is lost and from the point of view of the NMR diffusometry experiment, the sample becomes homogeneous, showing Gaussian diffusion.

5. Concluding remarks

We have investigated effects of finite gradient pulses in NMR diffusometry experiments using an emulsion system. For molecules confined to a pore during the diffusion time the pore size determined using the SGP approach is considerably smaller than the actual value when the pores size is of the same magnitude as $\sqrt{D\bar{\delta}}$. For molecules migrating through the porous system the appearance of the diffusion induced echo decay curves depend markedly on the gradient pulse length, everything else being equal. If the MSD during the gradient pulse is of the same order of magnitude as or larger than the pore size, the information of the pore size is gradually lost.

6. Experimental section

6.1. Materials and sample preparation

Heptane of p.a. quality was purchased from Merck. Aldrich supplied the surfactant Brij92. Tetramethyl ammonium chloride (TMACl) was purchased from Fluka Biochemica. The water was of Millipore-Q quality and heavy water was from Dr. Glaser, Basel. The composition of the concentrated emulsion was 95 wt% 0.2 M tetramethyl ammonium chloride in Millipore water as aqueous phase, 3.5 wt% heptane as oil phase and 1.5 wt% Brij 92 as surfactant. The water phase was added drop-wise to the oil phase (in which the surfactants were dissolved) in a glass tube containing seven glass beads while shaking on a mixer. When the emulsion became viscous the shaking was done by hand. The emulsion droplets were 3 μm in diameter and the film thickness between two adjacent droplets is about 10 nm.

6.2. NMR self-diffusion measurements

NMR experiments were performed on a 200 MHz Bruker DMX spectrometer equipped with a Bruker DIFF-25 gradient probe driven by a Bruker BAFPA-40 unit. The temperature was 25 °C. In the experiments the pulsed field gradient stimulated-echo technique was used [16]. δ was increased from 1.5 to 50 ms in 10 steps (with $\delta = 1.5, 2.2, 3.3, 4.8, 7.1, 10.5, 15.5, 22.9, 33.9$, and 50.0 ms) while $(\varDelta - \delta/3)$ was constant at 200 ms. The number of scans and the number of gradient steps were

32. By following the signal from tetramethyl ammonium ions, the influence of the pulse length for molecules confined to one pore is investigated, while water, because of its finite solubility in the oil film, is used to probe effects of finite pulses on molecules that perform pore-hopping [14]. For the latter case, keeping $(\varDelta - \delta/3)$ constant gives the same initial slope of the echo-attenuation for all values of δ. The echo-attenuations of Figs. 3 and 6 are normalized with the intercept obtained from a fit to the initial slope of the curves. The stability control of the emulsion was done with the following parameters: stimulated echo, $\delta = 2$ ms, $\varDelta = 100$ ms, and number of scans $= 4$.

6.3. Brownian dynamic simulations

The program package PGSE was used for simulating the spin echo attenuation for molecules confined to one pore. The procedure used is outlined in [17,18]. Simulations were performed for different pulse lengths, δ, as above. The input q-data are set close to the experimental values, the time step was 50 μs, the pore radii was set to 1.8 μm, and the number of spin trajectories 10^5. The diffusion coefficient for tetramethyl ammonium ion in a 0.2 M tetramethyl ammonium chloride water solution was experimentally determined to $D_0 = 9.8 \times 10^{-10}$ m^2 s^{-1}, and this value was used as input in the simulations.

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